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Nuclear Structure Studies in the Zirconium Isotopes with (d,p) and (d,t) Reactions*

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Angular distributions from (d,p) reactions in the zirconium isotopes are found to agree exceedingly well with distorted-wave Born approximation calculations; this agreement includes the absolute cross sections. These reactions are used to assign all levels excited by (d,p) reactions is Zr^{90}, Zr^{91}, Zr^{92}, Zr^{94}, and Zr^{96} to single-particle (S.P.) states. The summed cross sections for each S.P. state in each isotope are close to expectations. The "centers of gravity" of each S.P. state are located; all of these including the nonpaired levels from the (d,p) reaction on Zr^{91}, behave very smoothly as a function of A. The summed cross sections for states of $I=0$, 2, and 4 from $Zr^{91}(d,p)$ each also agree with theory. The ground state of Zr^{92} is found to be 78% $(d_{5/2})^2$. Results from (d,t) reactions on the even isotopes leading to states other than $d_{5/2}$ behave very anomalously. Attempts to explain them are discussed but more data are needed. The $Zr^{91}(d,t)$ reaction is used to obtain information on the ground state of Zr^{91}, and on the $3-$ state of Zr^{90}.

I. INTRODUCTION—EXPERIMENTAL

In a previous paper,[1] studies of nuclear structure in the zirconium isotopes using (d,p) and (d,t) reactions were reported. In that work, only rather thick and nonuniform ZrO targets of separated isotopes were available. The energy resolution from these was very poor and most proton groups [from the (d,p) reaction] were lost in oxygen background at most angles. As a result, the only useful angular distributions were those from a natural zirconium metal target, and the separated isotopes were only used to identify groups. This method worked reasonably well for the most abundant isotope ($A=90$), but gave very incomplete information for the less abundant isotopes. Nevertheless, the results

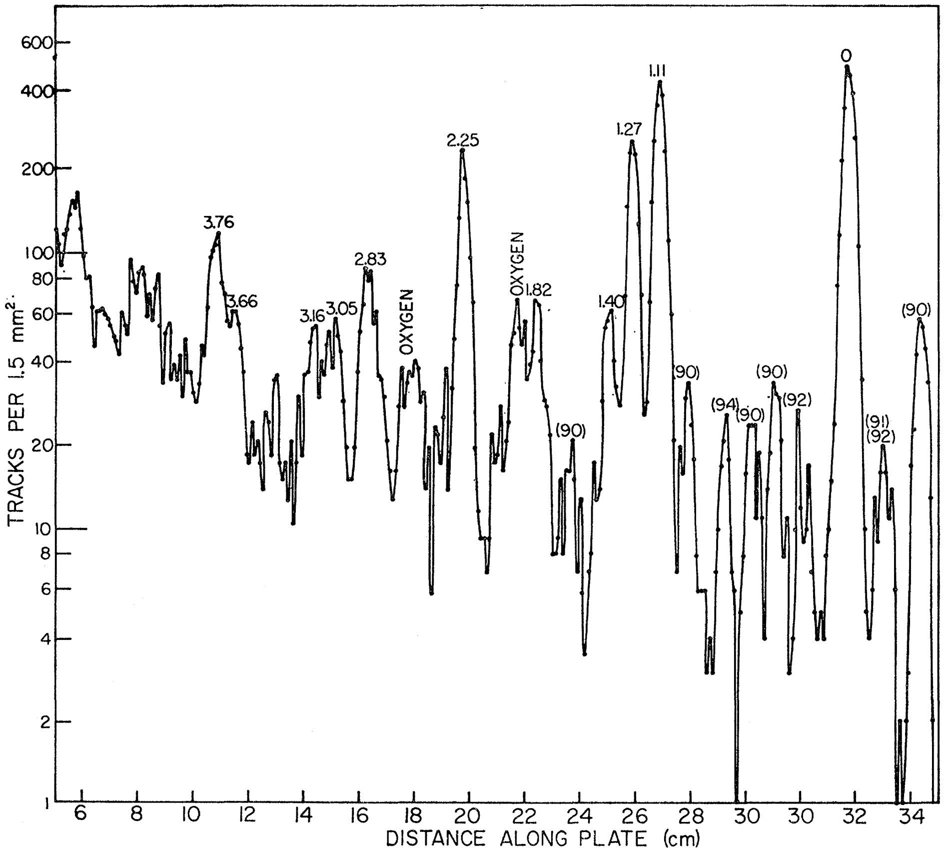

FIG. 1. Energy spectrum of protons from $Zr^{96}(d,p)Zr^{97}$. Angle of observation is 35°.

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[1] B. L. Cohen, Phys. Rev. **125**, 1358 (1962).

obtained were very interesting, and showed this region to be a fruitful one for further investigation.

Recently, metal foils highly enriched in the various zirconium isotopes became available.[2] In addition, distorted-wave Born approximation (DWBA) calculations of the (d,p) differential cross sections were obtained[3] and these proved highly successful in fitting the data. It was, therefore, decided to make a new experimental study.

The experimental method has been described previously.[4] Targets are bombarded with 15-MeV deuterons from the University of Pittsburgh 47-in. cyclotron; the reaction products are energy analyzed in passing through a 60° wedge-magnet spectrograph, and detected by the tracks they leave in photographic emulsions. The target thicknesses were about 5.5 mg/cm^2; this limits the energy resolution[5] to about 75 KeV at the most favorable angles, and to about 100 KeV at the least favorable angles.

The target thicknesses were somewhat nonuniform; this introduces a possible error of about 15% into relative cross sections from different isotopes. Since the relative cross sections for some transitions were accurately determined in Ref. 1, these were used to normalize target thicknesses. This procedure is not highly reliable due to errors in the two experiments, but as a result, all cross sections from Zr^{92} and Zr^{94} targets were increased by 10% over the direct determinations. Uncertainties in geometry limit the accuracy in *absolute* cross sections to about 15%.

For the Zr^{90} and Zr$^{96}(d,p)$ reactions, which are the most important for determining single-particle states, data were obtained at 5° intervals from 10° to 90°. For the other isotopes, (d,p) measurements were made at four angles chosen to give the maximum information on l-value assignments. For the (d,t) reactions, good measurements were obtained at only one angle. (See discussion below.)

A typical spectrum is shown in Fig. 1. It is from the Zr$^{96}(d,p)$ reaction, with the protons observed at 35°. The contamination from oxygen and other zirconium isotopes caused more difficulty in this reaction than in those on any of the other isotopes. Also, the energy range over which individual levels could be resolved was slightly shorter in the Zr^{96} data than in the data from the other targets.

II. COMPARISON WITH DWBA CALCULATIONS

As a basis for analyzing angular distributions, comparisons were made with DWBA calculations obtained from Satchler and collaborators.[3] The optical-model parameters, listed in footnote 3, were chosen on the basis of elastic-scattering data; no attempt was made to fit the (d,p) reaction data by varying these parameters.

Comparisons of calculated with experimentally measured angular distributions are shown in Fig. 2 for a few cases where the spins of the levels are known. In general, the agreement in the angular distributions is excellent. In the $l=2$ cases, the theoretically predicted variation of the angular distributions with Q value is also experimentally verified. Thus, the DWBA calculations probably can be considered to be a reliable tool for determining l values from angular distributions.

These calculations also turned out to be surprisingly accurate in predicting absolute cross sections. This is shown in Table I where the sum of all spectroscopic factors ($\sum S$) for transitions to the components of a given single-particle (S.P.) state are compared with the values expected from the analysis given in Ref. 1. The method of determining S will be discussed below (Sec. III), but for present purposes the important point is that it is proportional to the absolute cross sections predicted by the DWBA calculations. The agreement in Table I is within the combined uncer-

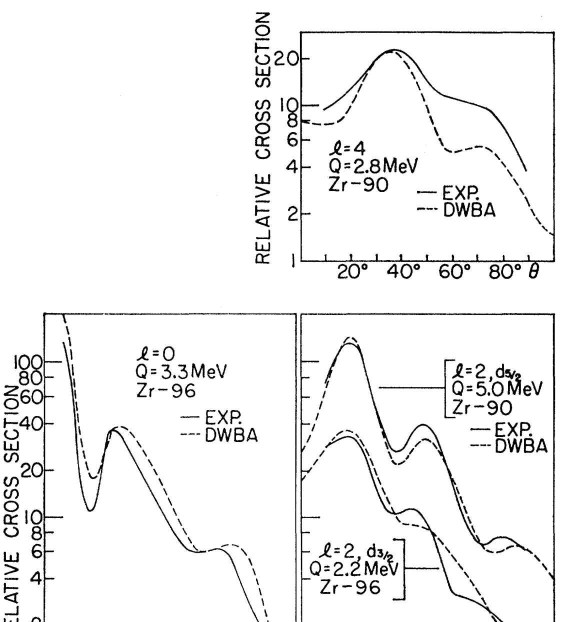

FIG. 2. Comparison between experimental angular distributions and calculations with distorted-wave Born approximation. There has been no normalization between experimental and theoretical curves, so that agreement indicates accuracy of DWBA calculations for absolute cross sections.

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[2] The authors are grateful to C. D. Goodman of Oak Ridge National Laboratory for making these targets available on loan.

[3] The authors are grateful to G. R. Satchler, R. M. Drisko, R. H. Bassel, and E. Halbert for performing these calculations. Optical model parameters used were: For deuteron: $V=90$, $r_0=1.23$, $a=0.64$, $W'=48$, $r_0'=1.18$, $a'=0.93$; for proton: $V=50$, $r_0=1.25$, $a=0.65$, $W'=50$, $r_0'=1.25$, $a'=0.47$. Surface derivative absorption, lower cutoff at 6.0 F (the last makes little difference except for $l=0$).

[4] B. L. Cohen, R. H. Fulmer, and A. L. McCarthy, Phys. Rev. **126**, 698 (1962).

[5] B. L. Cohen, Rev. Sci. Instr. **30**, 415 (1959).

TABLE I. Summary of results from (d,p) reactions on the various isotopes of zirconium. ΣS is the sum of spectroscopic factors for all levels belonging to that single-particle state from Tables II–VI, except for Zr^{91} where it is $(2j+1)^{-1}\Sigma S'$ (this last is not the usual definition, but is convenient here as it puts the Zr^{91} values on the same basis as those for the even isotopes). The last two columns are the excitation energy and the neutron binding energy for the "centers of gravity" of these states.

S.P. state	Target mass	ΣS Observed	ΣS Expected	E_j (MeV)	B.E. (MeV)
$d_{5/2}$	90	0.89	1.00	0	7.18
	91	0.82	0.83	0	8.62
	92	0.54	0.67	0	6.70
	94	0.30	0.33	0	6.43
$s_{1/2}$	90	0.96	1.0	1.55	5.63
	91	1.39	1.0	2.91	5.71
	92	1.13	1.0	1.15	5.55
	94	1.09	1.0	1.43	5.00
	96	0.98	1.0	0	5.57
$d_{3/2}$	90	1.00	1.0	2.70	4.48
	91	1.06	1.0	4.23	4.39
	92	1.01	1.0	2.40	4.30
	94	1.00	1.0	2.20	4.23
	96	0.83	1.0	1.37	4.20
$g_{7/2}$	90	0.97	1.0	2.70	4.48
	91	0.30	1.0	(4.8)	...
	92	0.92	1.0	2.4	4.30
	94	0.40	1.0	(2.6)	...
	96	0.85	1.0	1.64	3.93

tainties of the experimental determinations and the theoretical estimates.

III. LOCATION OF SINGLE-PARTICLE LEVELS

Since Zr^{90} and Zr^{96} are closed shell nuclei, (d,p) reactions on these are useful for locating single-particle states. The angular distributions of the proton groups observed in these reactions are shown in Figs. 3 and 4. By comparison with DWBA calculations, these are used to determine l, the angular momentum with which the neutron enters the nucleus. In the Zr^{90} (d,p) reaction, the determination of l values was reasonably straightforward. While the angular distributions have many features that are not easily explained (except perhaps as experimental error), the l assignment is usually fairly clear. One exception to this is the difficulty in differentiating between $l=2$ and $l=3$ at high excitation energy. Also, there are three groups (4.12, 4.29, and 4.52 MeV) which lead to unresolved multiplets and for which the angular distributions were not easily fit into any of the patterns shown in Fig. 3. In all these difficult cases, the levels are very weakly excited and the experimental errors are large so that they cannot be considered as serious obstacles to a consistent theory. They were tentatively assigned l values by the location of the first peak in their angular distributions. From the l value, j values are assigned from the shell-model states expected in this region. The only ambiguity is in assigning the $l=2$ states as $d_{3/2}$ or $d_{5/2}$. The ground state is known to be $d_{5/2}$; all other states are tentatively assigned as $d_{3/2}$; this will be discussed further below.

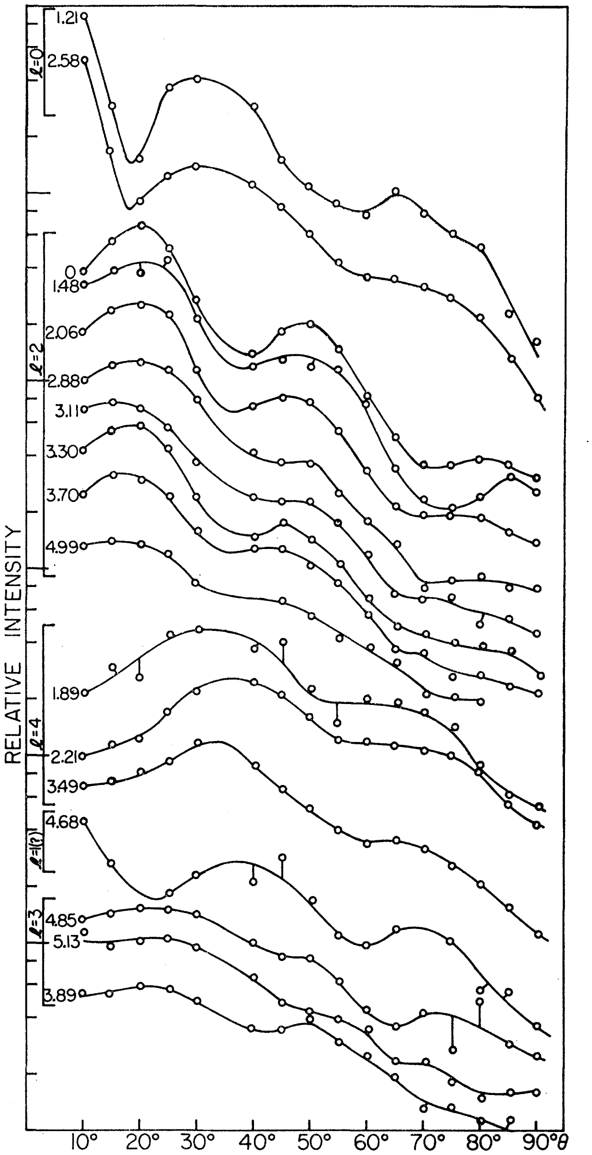

FIG. 3. Angular distributions for various groups excited in $Zr^{90}(d,p)$ reactions. Numbers attached to curves indicate excitation energy in MeV of final state in Zr^{91}. Brackets at left enclose angular distributions assigned to a given angular momentum transfer, l.

The data for $Zr^{90}(d,p)$ are summarized in Table II. The cross sections listed are those at the angle of the first maximum beyond 10° in the angular distribution; this angle, designated as θ_0, is approximately 30° for $l=0$, 17° for $l=2$, 30° for $l=4$, 20° for $l=3$, and 35° for $l=1$. The spectroscopic factor, S, is determined as the ratio of observed to calculated (using DWBA) cross sections at θ_0. The sum of S values for all levels belonging to a given single-particle state are listed in Table I; the fact that these sums agree with expectations gives confidence that essentially all levels are accounted for and properly assigned. The location of

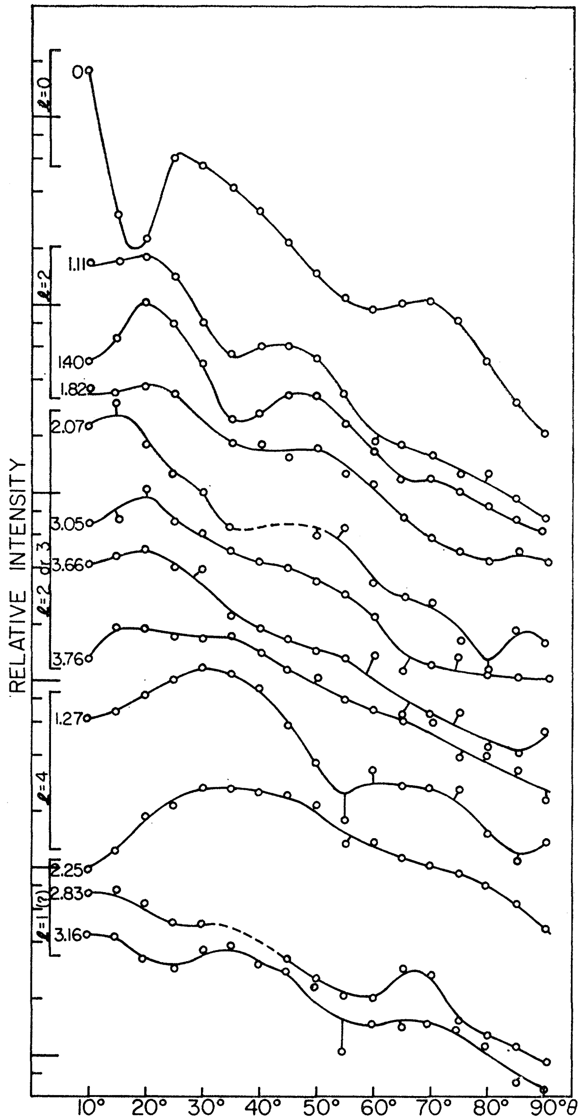

FIG. 4. Angular distributions for various groups excited in $Zr^{96}(d,p)$ reactions. See caption for Fig. 3.

the single-particle state, E_j, is taken as the "center of gravity" of all nuclear states of the proper j, weighting each according to the S value. These locations are listed in Table I. The S values of the various states and E_j are shown graphically in Fig. 5.

The procedure followed in the $Zr^{96}(d,p)$ reaction was generally analogous. The angular distributions are shown in Fig. 4 and the data are summarized in Table III. There was somewhat more difficulty in making l-value assignments here than in Zr^{90}, mostly due to the fact that Q values are lower and the difference between DWBA angular distributions for different l is less marked for lower Q. In general, no distinction could be made between $l=2$, and $l=3$ for the highly excited levels. In determining $\sum S$ and Ej, these were taken to be half $l=2$ and half $l=3$. There were also some difficulties from oxygen contamination in these angular distributions. All proton groups up to an excitation energy of 3.8 MeV are accounted for in Fig. 4. The results for Zr^{96} are shown in Table I and Fig. 5. These results and those for Zr^{90} are used by Cohen *et al.*[6] and analyzed by Cohen.[7]

TABLE II. Data from $Zr^{90}(d,p)Zr^{91}$ reactions. Methods of obtaining the various columns are described in the text. Asterisk denotes a group of unresolved levels. Values of l in parentheses are questionable. Bracketed rows are considered equally likely assignments for the same level.

Excit. en. (MeV)	l	$\sigma(\theta_0)$ (mb/sr)	j	S
0	2	15.3	$d_{5/2}$	0.89
1.21	0	4.5	$s_{1/2}$	0.72
1.48	2	0.38	$d_{3/2}$	0.029
1.89	4	0.17	$g_{7/2}$	0.062
2.06	2	6.4	$d_{3/2}$	0.45
2.21	4	1.56	$g_{7/2}$	0.52
2.35	(4)	0.14	$g_{7/2}$	0.05
2.58	0	1.32	$s_{1/2}$	0.24
2.88	2	1.30	$d_{3/2}$	0.078
3.11	2	1.82	$d_{3/2}$	0.105
3.30	2	2.7	$d_{3/2}$	0.15
3.49*	4	1.25	$g_{7/2}$	0.33
3.70	2	1.9	$d_{3/2}$	0.10
3.89*	3	1.01	$f_{7/2}$	0.042
4.12*	(4)	0.23	$g_{7/2}$	0.056
4.29*	(3)	0.62	$f_{7/2}$	0.025
4.52*	{2	0.81	$d_{3/2}$}	0.039
	3		$f_{7/2}$}	0.031
4.68*	1	0.30	$p_{3/2}$	0.022
4.85*	3	0.46	$f_{7/2}$	0.016
4.99*	(2)	0.95	$d_{3/2}$	0.042
5.13*	3	0.62	$f_{7/2}$	0.021

It is interesting to point out that in neither isotope was an $h_{11/2}$ level found, although it is expected in this region. The most likely candidate was the 2.25-MeV level in $Zr^{96}(d,p)$. The data for this level are compared with the DWBA calculations in Fig. 6. It is clear that the angular distribution strongly favors the $l=4$.

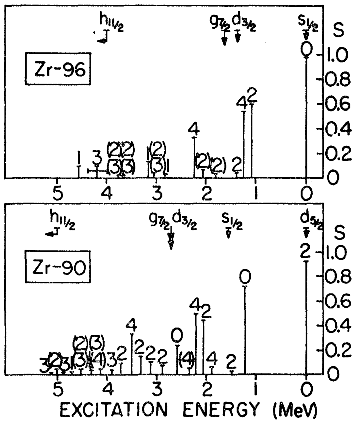

FIG. 5. S values from (d,p) reactions on Zr^{90} and Zr^{96}. Height of vertical line indicates S value, and figure above indicates l-value assignment. Arrows at top indicate "centers of gravity" of single-particle states as obtained from this work.

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[6] B. L. Cohen, R. H. Fulmer, A. L. McCarthy, and P. Mukherjee, Rev. Mod. Phys. **35**, 332 (1963).

[7] B. L. Cohen, Phys. Rev. **130**, 227 (1963).

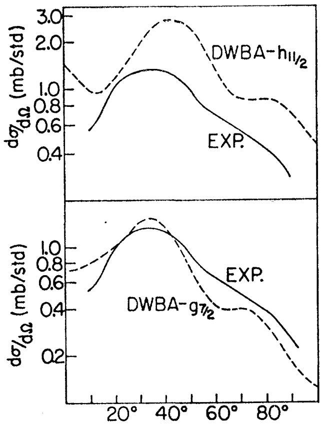

FIG. 6. Assignment of 2.25-MeV level from Zr^{96} (d,p). Upper figure shows comparison with DWBA if it is assigned as $h_{11/2}$ and lower figure shows this comparison if it is assigned as $g_{7/2}$. In the latter case, the contribution from the 1.27-MeV level has been subtracted from the DWBA prediction.

Furthermore, an $l=4$ assignment increases $\sum S$ for $g_{7/2}$ from 0.52 to 0.85 (which is close to the expected value), whereas an $l=5$ assignment would still account for only half of the $h_{11/2}$ state. It is also clear from a comparison of Figs. 1 and 6 that the cross sections for $h_{11/2}$ states are not expected to be small; they should be easily observable if they were in the region studied. Thus, we may conclude that the $h_{11/2}$ state lies above the region studied here, which puts it at excitation energy greater than 5.1 and 4.0 MeV in Zr^{90} and Zr^{96}, respectively. Another possibility is that the DWBA calculations for $l=5$ are grossly in error, but this seems unlikely in view of the excellent agreement for $l=0$, 2, and 4. A less probable explanation is that the $h_{11/2}$ state is split into a very large number of components so that none are strongly enough excited to be identified.

IV. LEVEL STRUCTURE IN ODD ISOTOPES OF ZIRCONIUM

It is clear from Figs. 2 and 3 that most of the information on l-value assignments can be obtained from measurements at a few key angles. Thus, to study the (d,p) reactions on Zr^{92} and Zr^{94}, spectra were obtained only at 9°, 17°, 30°, and 40°. In some cases of weakly excited levels, l assignments are difficult to make, but this situation was often not much better in the isotopes where complete angular distributions were measured, and the four angle method is certainly adequate for the principal single-particle levels which are strongly excited. On the whole, however, the results of the Zr^{92} and $Zr^{94}(d,p)$ reactions should be considered less reliable than those of Zr^{90} and Zr^{96}.

Data for the former are given in Tables IV and V, and the results are summarized in Table I. The values of θ_0 are the same as those for Zr^{90} for $l=0$, 2, and 4, θ_0 for $l=1$ was taken as 30° rather than 35°, and θ_0 for $l=3$ was taken as 17° rather than 20°. This would introduce some error, but the errors in $l=1$ and $l=3$ states are already large because of uncertain identifications, and furthermore, these are not used in any of the analysis.

TABLE III. Data from $Zr^{96}(d,p)Zr^{97}$ reaction. See caption for Table II.

Excit. en. (MeV)	l	$\sigma(\theta_0)$ (mb/sr)	j	S
0	0	5.6	$s_{1/2}$	0.98
1.11	2	10.0	$d_{3/2}$	0.60
1.27	4	1.8	$g_{7/2}$	0.54
1.40	2	1.9	$d_{3/2}$	0.11
1.82	(2)	0.78	$d_{3/2}$	0.042
2.07	(2)	0.60	$d_{3/2}$	0.031
2.25	4	1.35	$g_{7/2}$	0.33
2.83	(1)	1.1	$p_{3/2}$	0.08
3.05	2	1.00	$d_{3/2}$	0.046
	3		$f_{7/2}$	0.036
3.16	1	0.55	$p_{3/2}$	0.04
3.66	2	0.77	$d_{3/2}$	0.033
	3		$f_{7/2}$	0.026
3.76*	2	0.77		0.033
	3			0.026

TABLE IV. Data from $Zr^{92}(d,p)Zr^{93}$ reactions. See caption for Table II.

Excit. en. (MeV)	l	$\sigma(\theta_0)$ (mb/sr)	j	S
0	2	8.7	$d_{5/2}$	0.54
0.28	(?)	~0.10	(?)	⋯
0.96	0	5.3	$s_{1/2}$	0.91
1.45	2	5.3	$d_{3/2}$	0.38
1.64	(4)	0.27	$g_{7/2}$	0.11
1.94	0	1.07	$s_{1/2}$	0.21
2.08	(4)	0.97	$g_{7/2}$	0.42
2.32	4	0.23	$g_{7/2}$	0.09
2.50	2	4.1	$d_{3/2}$	0.24
2.78	2	3.6	$d_{3/2}$	0.21
3.02	4	1.08	$g_{7/2}$	0.30
3.19	2	0.71	$d_{3/2}$	0.38
3.29	2	0.50	$d_{3/2}$	0.028
3.41	1	1.26	$p_{3/2}$	0.117
3.64*	(2), (3), (4)	~0.43	⋯	⋯
3.78*	1	0.86	$p_{3/2}$	0.075
	2	1.43	$d_{3/2}$	0.069
4.03*	1	1.67	$p_{3/2}$	0.133
4.27*	1	0.35	$p_{3/2}$	0.028
4.40*	2	1.20	$d_{3/2}$	0.052
4.77*	1	1.65	$p_{3/2}$	0.119
5.00*	1	1.54	$p_{3/2}$	0.107

The only badly out-of-line result in Table I is $\sum S$ for the $g_{7/2}$ states in $Zr^{94}(d,p)$. These are the least certain data experimentally, and it may be hoped that further data will clarify the situation. The fact that $\sum S$ for the $d_{3/2}$ states in most isotopes is greater than for the $d_{5/2}$ in Zr^{90} is probably due to improper assignment of some weak $f_{7/2}$ (or even $d_{5/2}$) levels as $d_{3/2}$.

Zeroth-order predictions of states in the odd isotopes may be made by considering the various couplings of each single-particle state to each state of the target nucleus.[8,9] The even-parity states are shown and compared with experiment for Zr^{91} and Zr^{93} in Figs. 7 and

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[8] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences–National Research Council, Washington, 25 D. C., 1961).

[9] M. E. Bunker, B. J. Dropesky, J. D. Knight, and J. W. Starner, Phys. Rev. **127**, 844 (1962).

TABLE V. Data from $\mathrm{Zr}^{94}(d,p)\mathrm{Zr}^{95}$ reactions. See caption for Table II.

Excit. en. (MeV)	l	$\sigma(\theta_0)$ (mb/sr)	j	S
0	2	5.0	$d_{5/2}$	0.30
0.95	0	5.2	$s_{1/2}$	0.89
1.33	2	0.24	$d_{3/2}$	0.017
1.64	2	6.5	$d_{3/2}$	0.45
1.73	(2)	~0.7	$d_{3/2}$	~0.05
1.91*	2	1.20	$d_{3/2}$	0.078
2.03	4	0.30	$g_{7/2}$	0.106
2.29	(1)	1.13	$p_{3/2}$	0.124
2.40	2	1.37	$d_{3/2}$	0.080
2.48	(1)	0.22	$p_{3/2}$	0.024
2.65	(2)	0.79	$d_{3/2}$	0.044
2.75	4	0.87	$g_{7/2}$	0.26
2.87	2	1.77	$d_{3/2}$	0.099
3.03*	2	1.73	$d_{3/2}$	0.093
3.23	2	0.53	$d_{3/2}$	0.027
3.30*	0	0.49	$s_{1/2}$	0.109
3.38	(4)	0.10	$g_{7/2}$	0.039
3.54	{2	0.59	$d_{3/2}$	0.030}
	3		$f_{7/2}$	0.024
3.62	{2	0.59	$d_{3/2}$	0.030}
	3		$f_{7/2}$	0.024
3.68	{2	0.35	$d_{3/2}$	0.016}
	3		$f_{7/2}$	0.014
3.86	2	0.66	$d_{3/2}$	0.031
3.96	0	0.37	$s_{1/2}$	0.083

8, respectively. In Zr^{91}, the two $\frac{1}{2}^+$ states appear to arise from a mixing of the single particle $s_{1/2}$ state with the state $(\mathrm{Zr}^{90}-2.19\ \mathrm{MeV})\ d_{5/2}$. The higher $\frac{1}{2}^+$ states are too far away ($\gtrsim 1.5$ MeV) to mix with the single-particle state. For the $d_{3/2}$ and $g_{7/2}$, the mixing is very extensive, but the total number states in this energy region is about as expected.

In Zr^{93}, the number of observed states is less than half the number expected in the energy range over which the observed levels occur. This indicates that mixing of the single-particle state with nearby levels of the same spin and parity is not always appreciable.

The values of $\sum S$ in Table I are consistent with the simple picture that only the $d_{5/2}$ state is filling in the region between Zr^{90} and Zr^{96}. The uncertainties involved would allow up to about 10% of the $s_{1/2}$ and $d_{3/2}$ states to be filled in Zr^{96}. The relative cross sections for exciting the $d_{5/2}$ states in the various isotopes were more accurately determined in Ref. 1 (use of the DWBA calculations for Q-value corrections does not change those results detectably) than here; they are also consistent with this simple picture.

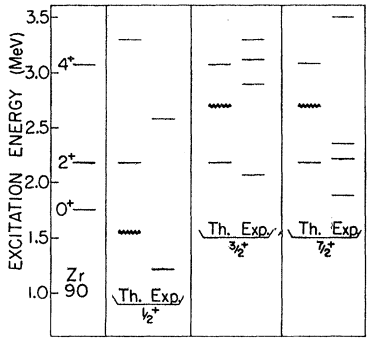

FIG. 7. Level structure of Zr^{91}. The theoretical levels are in zeroth order, assuming no interactions; they are obtained by coupling the single-particle levels to the states of Zr^{90}. The heavy lines are the single-particle states, and the others are states of the same spin and parity. Note suppressed zero.

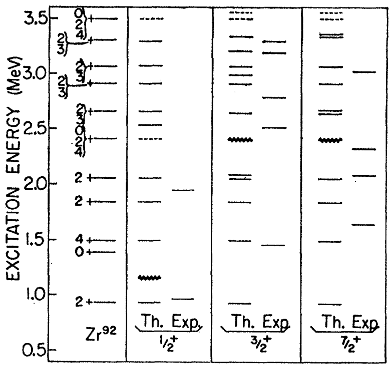

FIG. 8. Level structure of Zr^{93}. See caption for Fig. 7.

V. THE $\mathrm{Zr}^{91}(d,p)\mathrm{Zr}^{92}$ REACTION

The data from $\mathrm{Zr}^{91}(d,p)$ reactions are listed in Table VI. The order of listings deviates slightly from the usual order of increasing excitation energy to keep transitions of the same l together. The assignments of final-state spins are from other data for the low-lying states,[8,9] and from simple coupling considerations in all other cases except those to be discussed below. The quantity S' is defined as

$$S'=\frac{2I_f+1}{2I_i+1}S.$$

With this definition, the $\sum S'$ for all transitions in

TABLE VI. Data from $\mathrm{Zr}^{91}(d,p)\mathrm{Zr}^{92}$ reactions. See caption for Table II. Note that rows are not quite in order of increasing excitation energy. $\Sigma\, S'$ is the sum of S' for the rows enclosed by brackets.

Excit. en. (MeV)	l	I_f	$\sigma(\theta_0)$ (mb/sr)	S'	$\Sigma\, S'$ exp	$\Sigma\, S'$ theor
0	2	0	0.47	0.24	0.32	0.33
1.38	2	0	~0.2	~0.08		
0.94	2	2	2.50	1.11	1.70	1.67
1.88	2	2	0.98	0.36		
2.40	2	(2)	0.68	0.23		
1.50	2	4	7.1	2.9	2.9	3.00
2.07	0	2	0.99	0.30	2.79	2.00
2.66	0	2, 3	0.46	0.15		
2.91	0	3	3.45	1.22		
3.06	0	2, 3	1.44	0.50		
3.30	0	2, 3	1.67	0.62		
3.49	2	1–4	1.67	0.46	4.26	4.00
3.69	2	1–4	1.20	0.33		
3.81	2	1–4	1.65	0.44		
4.03	2	(2,3,4)	2.7	0.69		
4.14	2	(3,4)	3.7	0.93		
4.50	2	1–4	0.185	0.44		
4.65	2	1–4	0.96	0.23		
4.97	2	1–4	1.58	0.35		
5.30	2	1–4	1.80	0.39		
4.80	4	(6)	1.08	2.4	2.4	8.0
5.10	1	1–4	0.55	0.21	0.46	4.0
5.50	1	1–4	0.73	0.25		

which a neutron is inserted into a given single-particle state is equal to the number of vacancies in that state. If we assume that the ground state of Zr^{91} contains one $d_{5/2}$ neutron, $\sum S'$ for $d_{5/2}$, $s_{1/2}$, $d_{3/2}$, and $g_{7/2}$ states are 5, 2, 4, and 8, respectively. It is seen that if the $l=2$ states lying below 2.40 MeV and above 3.49 MeV are considered to be $d_{5/2}$ and $d_{3/2}$ transitions, respectively, the agreement is quite good for these states; also, it is not too bad for the $s_{1/2}$ states. It is clear that most of the $g_{7/2}$ states are missed (a minimum of 6 must be present).

Among the various states corresponding to $d_{5/2}$ stripping, $\sum S'$ for states going to a given I_f should be in the ratio of $2I_f+1$. If the 2.40-MeV state is assumed to have $I=2$, the agreement here is quite good. Among the $g_{7/2}$ states, only the $I=6$ state can have S' as large as that of the 4.80-MeV state ($\sum S'$ for $I=6$ should be 2.17 as compared to 1.83 for $I=5$, etc.) Similarly, the magnitude of S' precludes $I=1$ for the 4.03-MeV state, $I=1$ or 2 for the 4.14-MeV state and $I=2$ for the 2.91-MeV state if one takes into account the fact that the 2.07-Mev state is known to be $I=2$.

The "centers of gravity" of the various single-particle states are listed in Table I (as E_j), and the binding energy for a neutron in each state, calculated from these and ground-state Q values,[10] is listed in the last column of Table I. It is seen that the data from Zr^{91} fits in quite smoothly with the data from the even isotopes. This would indicate that the average interaction energy between nonequivalent neutrons is the same whether or not one of them is paired.

The S values for the ground, 0.94-, and 1.50-MeV states agree well with the results of Martin, Sampon, and Preston.[11] The result for the ground-state transition also agrees within experimental error with the determinations from both this reaction and the $Zr^{92}(d,t)$ reaction in Ref. 1. The weighted mean of these is $S'=0.26$. This implies that $S(d,p)$ for the $Zr^{92}(d,p)$ reaction should be 0.74 rather than 0.59 as given in Table I [after taking the ratio to $Zr^{90}(d,p)$] or 0.67 as given in Ref. 1. There must, therefore, be one or more $d_{5/2}$ states at higher excitation.

VI. RESULTS FROM (*d,t*) REACTIONS ON EVEN ISOTOPES

Unfortunately, useful data on (d,t) reactions was obtained at only one angle, namely, 47°. By the time this was realized, the targets had been returned and the cyclotron was about to begin a long shutdown, so that it was not deemed practical to delay publication until more data were obtained.

TABLE VII. Results from (d,t) reactions on even isotopes of zirconium. The $E(d,p)$, j, and $S(d,p)$ are from Tables II–V, under the assumption that the levels observed are the same as those observed in the (d,p) reactions. The evidence for this is the correspondence in energies. The methods of obtaining $S(d,t)$ are described in the text; note that they are not highly quantitative. The last column is the ratio of the previous two columns.

$E(d,t)$ (MeV)	$E(d,p)$ (MeV)	j	$\sigma(d,t)$ (mb/sr)	$S(d,t)$	$S(d,p)$	$S(d,p)/S(d,t)$
⟨A. Results for $Zr^{92}(d,t)Zr^{91}$ reactions⟩						
0	0	$\frac{5}{2}$	1.20	1.56	0.89	0.56
1.22	1.21	$\frac{1}{2}$	0.186	0.15	0.72	4.9
1.91	1.89	$\frac{7}{2}$	$\sim$0.008	$\sim$0.05	0.062	1.3
2.07	2.06	$\frac{3}{2}$	0.057	0.10	0.45	4.5
2.19	2.21	$\frac{7}{2}$	0.059	0.43	0.52	1.2
⟨B. Results for $Zr^{94}(d,t)Zr^{93}$ reactions⟩						
0	0	$\frac{5}{2}$	2.31	3.41	0.54	0.16
0.94	0.96	$\frac{1}{2}$	0.36	0.32	0.91	2.8
1.46	1.45	$\frac{3}{2}$	0.074	0.15	0.38	2.6
1.65	1.64	$(\frac{7}{2})$	0.034	0.27	0.11	0.33
1.91	1.94	$\frac{1}{2}$	0.056	0.056	0.21	3.6
2.00	···	$(\frac{5}{2})$	0.073	0.15	$\gtrless$0.02	$\gtrless$0.14
2.08	2.08	$(\frac{7}{2})$	$\sim$0.024	$\sim$0.20	0.42	2.1
2.20	···	$(\frac{5}{2})$	$\sim$0.033	$\sim$0.070	$\gtrless$0.02	$\gtrless$0.35
2.33	2.32	$\frac{7}{2}$	0.019	0.16	0.09	0.56
2.48	2.50	$\frac{3}{2}$	0.12	0.28	0.24	0.86
⟨C. Results from $Zr^{96}(d,t)Zr^{95}$ reactions⟩						
0	0	$\frac{5}{2}$	3.76	5.75	0.30	0.052
0.96	0.95	$\frac{1}{2}$	0.200	0.18	0.89	5.0
1.33	1.33	$\frac{3}{2}$	0.020	0.039	0.017	0.44
1.65	1.64	$\frac{3}{2}$	0.042	0.084	0.45	6.4
1.75	1.73	$\frac{3}{2}$	0.015	0.032	0.05	1.6
1.92	1.91	$\frac{3}{2}$	0.094	0.20	0.078	0.39
2.03	2.03	$\frac{7}{2}$	0.047	0.41	0.106	0.26
2.30	2.29	$p_{3/2}$(?)	0.073	0.165	0.124	0.75
2.40	2.40	$\frac{3}{2}$	0.055	0.127	0.080	0.63
2.67	2.65	$\frac{3}{2}$	0.030	0.072	0.044	0.61
2.77	2.75	$\frac{7}{2}$	0.146	1.42	0.26	0.18
2.88	2.87	$\frac{3}{2}$	0.054	0.134	0.099	0.75
3.05	3.03	$\frac{3}{2}$	0.030	0.076	0.093	1.2

The results are summarized in Table VII. The excitation energies measured in the (d,t) data agree within experimental error with those measured in the (d,p) reactions. This gives one confidence that the same levels are being observed in the two reactions (except, perhaps, in very few cases) and, thus, gives the spins and parities. The differential cross sections are listed in column 4. In order to obtain the spectroscopic factors, S, corrections must be made for Q-value dependence, and the proper normalizations for each l must be introduced. The Q-value dependence was taken to be the same (with sign reversed) as in the DWBA calculations for (d,p) reactions. This turns out to be 18% per MeV which is the same correction as was used in Ref. 1. The normalization for $l=2$ was obtained by taking the S value for the ground-state transition to be the same as in Ref. 1 (except for Zr^{92} where the ground-state S value from the last section is used); it was then assumed that the normalizing factor for $l=0$ and $l=4$ transitions is $\frac{1}{2}$ and 4 times as large, respectively, as that for $l=2$. These are the factors found experimentally in the Sn region,[12] and are in general agreement with theoretical expectations.

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[10] R. Patell, M. S. thesis, University of Pittsburgh, 1963 (unpublished).

[11] H. J. Martin, M. B. Sampon, and R. L. Preston, Phys. Rev. **125**, 942 (1962).

[12] B. L. Cohen and R. E. Price, Phys. Rev. **121**, 1441 (1961).

The S values obtained by this procedure are listed in column 5 of Table VII and they are compared with the S values from (d,p) reactions leading to the same levels in columns 6 and 7. The uncertainty in $S\ (d,t)$ for $l=2$ levels is about 30% due to the fact that data at only a single angle are used. The usual procedure is to average over several angles, so that the ground-state S values, taken from Ref. 1, have an uncertainty of only about 10%. For the $l=0$ and $l=4$ levels, there is an additional uncertainty of about 40% due to uncertainties in the normalization. Two levels of Zr^{93} are found in (d,t) reactions but not in (d,p); they are tentatively assigned as $\frac{5}{2}^+$, as only these have a small ratio of $S(d,p)/S(d,t)$.

The results for $\mathrm{Zr}^{92}(d,t)$ give an indication of how the 22% of the Zr^{92} ground state which is not $(d_{5/2})^2$ is distributed. It is very roughly 14% $(g_{7/2})^2$, 5% $(s_{1/2})^2$, and 3% $(d_{3/2})^2$; these values should be considered very tentative in view of the discussion to follow.

The results in Table VII for the Zr^{94} and $\mathrm{Zr}^{96}(d,t)$ reactions show two anomalies. Firstly, the values of $S(d,p)$ and $S(d,t)$ are almost completely uncorrelated, contrary to theoretical expectations; and secondly, the sums of the S values, which should be $(A-90)$ where A is the target mass, are considerably more than this.

(1) Correlations between $S(d,p)$ and $S(d,t)$. As an example of the reason why correlations are expected between $S(d,p)$ and $S(d,t)$ for levels of the same spin and parity, consider excitation of $\frac{3}{2}^+$ states in Zr^{95} with the following configurations assumed dominant in stripping and pickup reactions.

$$\mathrm{Zr}^{94}(0^+)-A^{1/2}(d_{5/2})^4+\cdots,$$
$$\mathrm{Zr}^{95}(\tfrac{3}{2}^+)-B^{1/2}(d_{5/2})^4d_{3/2}+\cdots,$$
$$\mathrm{Zr}^{96}(0^+)-C^{1/2}(d_{5/2})^6+D^{1/2}(d_{5/2})^4(d_{3/2})^2+\cdots.$$

From these, one calculates[13]

$$S(d,p)=AB,$$
$$S(d,t)=2BD.$$

Thus, $S(d,p)/S(d,t)=A/2D$, so that, although the coefficient B is different for each $\frac{3}{2}^+$ state of Zr^{95}, this ratio remains the same.

It is not difficult to imagine other parts of the configurations than those listed above as contributing to these reactions. This might explain the lack of correlation between $S(d,p)$ and $S(d,t)$ for levels weakly excited in both reactions, but only with the greatest difficulty can one explain the small value of $S(d,t)$ for the 1.65-MeV state,[14] for which $S(d,p)=0.45$.

(2) Sum of S Values. Even granting the maximum error in ground-state cross sections in $\mathrm{Zr}^{94}(d,t)$, the sum of $S(d,t)$ for all other states should be 0.9, whereas the experimental sum is 1.7; and in $\mathrm{Zr}^{96}(d,t)$ the corresponding numbers are 0.8 as opposed to 2.7. It is very difficult to believe that the normalization errors are that large.

If one assumes that somehow the discrepancy is due to normalization difficulties and, therefore, renormalizes, Table VII, Part C would indicate that the $d_{5/2}$ state is only $\frac{2}{3}$ full in Zr^{96}. This would then destroy its closed-shell behavior. However, the evidence for closed-shell behavior is very strong, including a very characteristic dependence of 2^+ collective state energy on mass number,[1] a large discontinuity in ground-state masses (see Table I), the absence of low-lying $d_{5/2}$ states in Zr^{97} (see Table III), etc. In addition, the fine agreement with $\sum S(d,p)$ in Table I would be destroyed.

Over the region studied, there is no indication of a slackening in (d,t) transitions as one goes to higher excitation energies. Thus, the problems discussed here would almost surely be compounded if the energy range of the experiment were extended.

One possible explanation is that "pickup" of $g_{9/2}$ (and $p_{1/2}$) neutrons is playing an important role. In order to reach $\frac{3}{2}^+$ and $\frac{7}{2}^+$ states, a recoupling is needed, so this would be a "forbidden" process. Furthermore, one does not expect to excite $g_{9/2}$ holes much below 4-MeV excitation energy.

Another possible explanation is that some process other than neutron pickup plays a role in (d,t) reactions. A knockout process might be a possibility. A compound-nucleus process would seem to be excluded by the observation that the general intensity of tritons decreases by a factor of 3 between 47° and 90°. Furthermore, a triton is an unlikely candidate for emission in a compound-nucleus process.

In view of the very surprising results, and in view of the fact that the (d,t) data are rather sparse and even the (d,p) data from Zr^{92} and Zr^{94}, which play a key role in the interpretations, leave something to be desired, it is clear that a great deal of further work is needed. Thinner isotopic targets have been ordered, and these matters will be reinvestigated in much greater detail when the cyclotron returns to operation.

VII. RESULTS FROM $\mathrm{Zr}^{91}(d,t)\mathrm{Zr}^{90}$ REACTIONS

The results from the $\mathrm{Zr}^{91}(d,t)$ reaction are given in Table VIII. The S' are calculated as in the last section, and the proton configurations for the first three states are taken from calculation by Talmi and Unna.[15]

The very small cross section for exciting the 1.75-MeV state indicates that the ratio of $(p_{1/2}{}^2)_0$ and $(g_{9/2}{}^2)_0$ in the ground states of Zr^{90} and Zr^{91} are almost exactly identical. The straightforward interpretation of the cross section to the 2.21-MeV state is that the ground-state configuration of Zr^{91} includes 5.5% of $\{(g_{9/2}{}^2)_2d_{5/2}\}_{5/2}$. This would also have the effect of de-

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[13] J. B. French, in *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960), p. 916.

[14] The (d,t) cross section reported for this level in Ref. 1 was much larger. This was found to be due to interference from the 2.21-MeV level from $\mathrm{Zr}^{91}(d,t)$, which occurs at the same energy. The target used in Ref. 1 contained only 2.6% Zr^{96}.

[15] I. Talmi and I. Unna, Nucl. Phys. **19**, 225 (1960).

TABLE VIII. Results for $Zr^{91}(d,t)Zr^{90}$ reactions. The spins and parities of the final states (of Zr^{90}) are known from other sources. The X indicates the ratio of the normalization for the unknown l of the 2.77-MeV state to that of $l=2$.

$E(d,t)$	I_f	$\sigma(d,t)$ (mb/sr)	S'	Proton configuration
0	0^+	0.95	1.00	$(0.75)^{1/2}p_{1/2}{}^2+(0.25)^{1/2}g_{9/2}{}^2$
1.75	0^+	0.004	0.005	$(0.25)^{1/2}p_{1/2}{}^2-(0.75)^{1/2}g_{9/2}{}^2$
2.21	2^+	0.036	0.055	$(g_{9/2})^2$
2.77	3^-	0.041	$0.066X$	

creasing the normalization for $l=2$ (d,t) reactions in the other zirconium isotopes by 5.5%; for example, the number of $d_{5/2}$ particles in Zr^{96} would be reduced from 5.75 to 5.44 (±10% in each case).

Another possible explanation for the (d,t) reaction to the 2.21-MeV state is to assume that it contains a fraction f of $(d_{5/2}g_{9/2}{}^{-1})$ in its neutron configurations and that the reaction proceeds by $g_{9/2}$ pickup. This would increase S' by a factor of 4, whence it would require $f\simeq 0.22$. This is unexpectedly large.

The most likely explanation for the excitation of the 2.77-MeV level is that its configuration contains a fraction of F of $(d_{5/2}p_{1/2}{}^{-1})$, so that the process proceeds by a pickup of a $p_{1/2}$ neutron. One then expects $X\simeq\frac{1}{2}$, so that $F\simeq 3\%$.

However, in view of the difficulties discussed in Sec. VI, judgment should perhaps be reserved on any conclusions from weak transitions in (d,t) reactions.

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Elastic Scattering of 1.2-BeV/c Muons from Hydrogen*

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The absolute cross section for the elastic scattering of negative muons from protons was measured over a range of momentum transfers of 450 to 850 MeV/c. The muon beam was formed by decay in flight of Bevatron produced pions and was separated from the pion beam electronically by using four gas-filled threshold Čerenkov counters. A total of 3×10^8 muons were incident on two large liquid hydrogen targets in tandem and gave a total of 56±9 acceptable scattering events, as compared to 48 predicted by the Rosenbluth formula for electromagnetic scattering from protons. A χ-square analysis of the scattered events gave agreement at the 75% level for the angular distribution of the data and the theoretical predictions, and gave with 95% confidence $\Lambda^{-1}\leq 0.16$ F, where Λ^{-1} is the conventional breakdown parameter. Hence, in this experiment, the behavior of muons scattered from protons at large momentum transfers is indistinguishable from that of electrons.

I. INTRODUCTION

THE high-energy scattering of muons in nuclear matter has been the object of many experimental investigations with the hope of uncovering a fundamental difference between muons and electrons. Prior to about 1958 these investigations showed a wide range of results with respect to the appropriate form of the electromagnetic cross section and there appeared to be a strong possibility that a large anomaly existed in the muon interaction, which might be due to a nonelectromagnetic interaction or a breakdown in quantum electrodynamics for the muon.[1] Since 1958 several new experiments have been done with both cosmic-ray and accelerator-produced muons.[2–8] These experiments have covered a wide range of incident muon energies (20–2000 MeV) and momentum transfers (20–400 MeV/c), and have used various target nuclei (carbon, lead, and nuclear emulsions). With one exception,[8] they have all

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[1] For a summary of high-energy muon experiments prior to 1958 see G. N. Fowler and A. W. Wolfendale, Progr. Elem. Particle Cosmic Ray Phys. **4**, 123 (1958).

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